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Synchronization of a chaotic map in the presence of common noise

M. K. Ali
Department of Physics, The University of Lethbridge, Lethbridge, Alberta T1K 3M4, Canada

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Synchronization of a model chaotic map is studied using the method of variable feedback. We report that when the driver and the response systems of the model map are subjected to common noise, the noise does not forbid synchronization. [S1063-651X(97)03604-0]

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Synchronization of chaos, hyperchaos, and spatiotemporal chaos has received much attention [1] in recent years. One of the methods used for synchronization is known as the method of variable feedback [2]. Synchronization of chaos in the presence of noise is important from practical considerations. We present results for the case when the driver and response systems of the feedback method are subjected to common noise. In order to illustrate the method and our main results, we consider the logistic map as our model. The purpose is to synchronize the chaotic map

$$x_{n+1}=4\,x_n(1-x_n) \tag{1}$$

with its replica

$$y_{n+1}=4\,y_n(1-y_n), \tag{2}$$

where n is the discrete time. In the method of variable feedback, this is achieved by introducing a feedback function $G(x_n,y_n)$ and iterating Eqs. (1) and (3),

$$y_{n+1}=4\,y_n(1-y_n)+G(x_n,y_n), \tag{3}$$

simultaneously starting from different initial values of x_0 and y_0. The systems described by Eqs. (1) and (3) are respectively called the driver and response systems. When synchronization is achieved after the transition time τ (value of n), $G(x_n,y_n)$ goes to 0 and $x_n=y_n$. The feedback function $G(x_n,y_n)$ is not unique—many different feedback functions can synchronize the map. For example $G(x_n,y_n)=(y_n-x_n)(2x_n-1)$ and $G(x_n,y_n)=(y_n-x_n)(2x_n-1)^3$ synchronize the logistic map of course with different transition time. The multitude of feedback functions that synchronize a given map are found by trial and error. In order to synchronize the logistic map with its replica in the presence of common noise, we add common random numbers $W\eta$ to these two systems at every iteration, as described by the equations

$$x_{n+1}=4\,x_n(1-x_n)+W\eta, \tag{4}$$

$$y_{n+1}=4\,y_n(1-y_n)+\beta[G(x_n,y_n)+W\eta], \tag{5}$$

where W is the strength of the noise and η is a equally distributed random number in the interval $-1\leqslant\eta\leqslant1$. Values of $W\eta$ are used that yield bounded solutions $0\leqslant x_{n+1}\leqslant1$ of the logistic map. If, during iterations, y_{n+1} falls beyond the interval (0,1), β is adjusted in $0\leqslant\beta\leqslant1$, so that we have $0\leqslant y_{n+1}\leqslant1$. Recently, Maritan and Banavar [3] reported that they could synchronize the logistic map without the feedback term that is by simply iterating the equations

$$x_{n+1}=4\,x_n(1-x_n)+W\eta, \tag{6}$$

$$y_{n+1}=4\,y_n(1-y_n)+W\eta. \tag{7}$$

TABLE I. Synchronization of chaotic trajectories of the logistic map using feedback function $fd1$ in the presence of common noise. Digits=3 and Digits=15 are the numerical precisions set in the MAPLE program, W is the strength of the noise term, and n is the iteration number. Trajectories started from $x_0=0.127$ and $y_0=0.713$ are seen to synchronize to the set precisions after 11 and 15 iterations for $W=0$ and 1, respectively. A similar trend in transition times is observed with other random initial state vectors x_0 and y_0.

		$n=11$, $W=0$		$n=15$, $W=1.0$	
x_n,y_n	$n=0$	Digits=3	Digits=15	Digits=3	Digits=15
x_n	0.127	0.235	0.984 914 415 149 068	0.093	0.503 143 004 925 717
y_n	0.713	0.235	0.984 914 415 149 068	0.093	0.503 143 004 925 717

TABLE II. Synchronization of chaotic trajectories of the logistic map using feedback function $fd2$ in the presence of common noise. Digits=3 and Digits=15 are the numerical precisions set in the MAPLE program, W is the strength of the noise term, and n is the iteration number. Trajectories started from $x_0=0.127$ and $y_0=0.713$ are seen to synchronize to the set precisions after 47 and 128 iterations for $W=0$ and 1, respectively. A similar trend in transition times is observed with other random initial state vectors x_0 and y_0.

		$n=47$		$n=128$	
		$W=0$		$W=1.0$	
x_n, y_n	$n=0$	Digits=3	Digits=15	Digits=3	Digits=15
x_n	0.127	0.0	0.007 224 728 906 920	0.921	0.681 224 990 446 831
y_n	0.713	0.0	0.007 224 728 906 920	0.921	0.681 224 990 446 831

Longa, Curado, and Oliveira [4] and Pikovsky [5] showed that, if the calculations are done with higher precisions, the transition time τ increases exponentially and the procedure soon becomes practically impossible to implement. For example, with a precision of 15 decimal places (digits=15 in a MAPLE program), $W=0.3$ requires 10^{12} iterations. Clearly, the application of common noise alone for synchronization of chaotic attractors has inherent difficulties. In this work we do not use the common noise to synchronize the logistic map but show that, when common noise is added, the noise does not affect exact synchronization by the method of variable feedback. Here we present results obtained with the two feedback functions $fd1$ and $fd2$, where

$$fd1=G(x_n,y_n)=4\,(y_n-x_n)(2x_n-1), \tag{8}$$

$$fd2=G(x_n,y_n)=4\,(y_n-x_n)(2x_n-1)^3. \tag{9}$$

It can be seen from Tables I and II that, for given x_0, y_0, and W, the iterated values of x_n and y_n depend on the precision used. However, for every precision the trajectory of the response system faithfully follows that of the driving system. These tables also show that the feedback function $fd1$ has a shorter transition time than $fd2$. The effect of the common noise is to increase the transition time of the two feedback functions, but not to eliminate synchronization.

Since a given map can be synchronized by very many feedback functions, a general method for finding all of them is not currently available. Once we have found a feedback function by trial and error or otherwise, common noise can be added to the procedure. On the basis of our model calculation, we conjecture that common noise does not forbid synchronization by the method of variable feedback.

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[1] E. Ott, C. Grebogi and J. A. York, Phys. Rev. Lett. **64**, 1196 (1990); L. Pecora and T. Carroll, *ibid.* **64**, 821 (1990); L. Kocarev and U. Parlitz, *ibid.* **77**, 2206 (1996); K. Kaneko, Physica D **55**, 368 (1992); T. Shinbrot, Adv. Phys. **44**, 73 (1995).

[2] K. Pyragas, Phys. Lett. A **170**, 421 (1992); Y. Nagai and Y. Lai, Phys. Rev. E **51**, 3842 (1995); H. D. I. Abarbanel, N. F. Rulkov, and M. M. Sushchik, *ibid.* **52**, 214 (1995).

[3] A. Maritan and J. R. Banavar, Phys. Rev. Lett. **72**, 1451 (1994).

[4] Lech Longa, Evaldo M. F. Curado, and Fernando A. Oliveira, Phys. Rev. E **54**, 2201 (1996).

[5] A. S. Pikovsky, Phys. Rev. Lett. **73**, 2931 (1994).